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Influence of Wind Speed on Airship Dynamics

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A new formulation of the equations of motion of an airship is derived to allow the analysis of the wind influence on the airship dynamics. Initially, the equations of motion are written via the Lagragian approach, considering the three kinetic energy terms associated with 1) the energy of the vehicle motion itself, 2) the energy of the air around the airship due to the relative velocities, and 3) the energy added to the buoyancy air. After that, the equations of motion are translated into Newton's second law formulation, yielding a new term, wind-induced force and torque. When the airship geometry approximated by an ellipsoid of revolution is considered the wind-induced terms are then explicitly derived, and their contribution on the longitudinal and lateral dynamics of the airship motion is analyzed. The results are illustrated using the model of a real airship, considering a given range of wind speed and a constant low airspeed.

Nomenclature

ABC	=	local frame, centered in O nearly equal to center of volume (CV) (Fig. 1)
$\boldsymbol{A}_h \in \Re^{4\times 4}$, $\boldsymbol{B}_h \in \Re^{4\times 4}$	=	matrices of linearized lateral dynamic equations
$\boldsymbol{A}_v \in \Re^{4\times 4}$, $\boldsymbol{B}_v \in \Re^{4\times 3}$	=	matrices of linearized longitudinal dynamic equations
$\boldsymbol{a}_g$	=	gravity acceleration in the inertial, Earth frame, north, east, down (NED) frame, $[0, 0, g]^T \in \Re^{3\times 1}$
$\boldsymbol{C}$	=	center of mass of the airship, c.g.
$\bar{F}$	=	total external force and torque applied on airship, $[\boldsymbol{F}^T, \boldsymbol{T}^T]^T \in \Re^{6\times 1}$
$\overline{\boldsymbol{F}_a} \in \Re^{6\times 1}$	=	aerodynamic force and torque
$\overline{\boldsymbol{F}_g} \in \Re^{6\times 1}$	=	gravity force and torque
$\overline{\boldsymbol{F}_k} \in \Re^{6\times 1}$	=	kinematics force and torque
$\overline{\boldsymbol{F}_p} \in \Re^{6\times 1}$	=	propulsion force and torque
$\overline{\boldsymbol{F}_w} \in \Re^{6\times 1}$	=	wind-induced force and torque
$\boldsymbol{H} \in \Re^{3\times 1}$	=	angular momentum vector
h_0	=	reference altitude for linearization
$I_3 \in \Re^{3\times 3}$	=	identity matrix
$\boldsymbol{J} \in \Re^{3\times 3}$	=	inertia matrix of the airship
$\boldsymbol{J}_a \in \Re^{3\times 3}$	=	apparent inertia matrix of airship, $\boldsymbol{J} + \boldsymbol{J}_v$
$\boldsymbol{J}_B \in \Re^{3\times 3}$	=	inertia matrix of the buoyancy air
$\boldsymbol{J}_{Ba} \in \Re^{3\times 3}$	=	apparent inertia matrix of the buoyancy air, $\boldsymbol{J}_B + \boldsymbol{J}_v$
$\boldsymbol{J}_v \in \Re^{3\times 3}$	=	virtual inertia matrix
$\boldsymbol{M} \in \Re^{3\times 3}$	=	inertial mass matrix of the airship, $m\boldsymbol{I}_3$
$\boldsymbol{M}_a \in \Re^{3\times 3}$	=	apparent mass matrix of airship, $m\boldsymbol{I}_3 + \underline{\boldsymbol{M}_v}$
$\boldsymbol{M}_{ah} \in \Re^{3\times 3}$	=	apparent mass matrix (obtained from $\overline{\boldsymbol{M}_a}$) for the lateral motion
$\boldsymbol{M}_{av} \in \Re^{3\times 3}$	=	apparent mass matrix (obtained from $\overline{\boldsymbol{M}_a}$) for the longitudinal motion
$\boldsymbol{M}_B \in \Re^{3\times 3}$	=	inertial mass matrix of the buoyancy air, $m_B\boldsymbol{I}_3$
$\boldsymbol{M}_{Ba} \in \Re^{3\times 3}$	=	apparent mass matrix of the buoyancy air, $m_B\boldsymbol{I}_3 + \boldsymbol{M}_v$
$\underline{\boldsymbol{M}_v} \in \Re^{3\times 3}$	=	virtual mass matrix
$\overline{\boldsymbol{M}_a} \in \Re^{6\times 6}$	=	generalized apparent mass matrix of airship with inertias and masses
$\overline{\boldsymbol{M}_v} \in \Re^{6\times 6}$	=	generalized virtual mass matrix with inertias and masses
$m \in \Re$	=	airship scalar mass
$m_B \in \Re$	=	buoyancy mass, product of air density by airship volume, ρV
$\boldsymbol{O}$	=	origin of local frame, CV = center of buoyancy (CB)
$\boldsymbol{OC}$	=	vector from CV to c.g., $\boldsymbol{C} - \boldsymbol{O} = [a_x, 0, a_z]^T \in \Re^{3\times 1}$
$\boldsymbol{p} \in \Re^{3\times 1}$	=	linear momentum vector
$\boldsymbol{S} \in \Re^{3\times 3}$	=	transformation matrix from NED frame to ABC frame
$\boldsymbol{V}$	=	vector of inertial airship speed represented in ABC frame, $[u, v, w]^T \in \Re^{3\times 1}$
V_t	=	true airspeed, $\sqrt{(u_a^2 + v_a^2 + w_a^2)}$
V_{t0}	=	reference airspeed, for linearization
$\boldsymbol{V}_a$	=	vector of linear airspeed, $\boldsymbol{V} - \boldsymbol{V}_w \in \Re^{3\times 1}$
$\boldsymbol{V}_w$	=	vector of wind linear speed represented in ABC frame, $[u_w, v_w, w_w]^T \in \Re^{3\times 1}$
$W \in \Re$	=	kinetic energy
$W^c \in \Re$	=	kinetic energy referred to the c.g.
$W^0 \in \Re$	=	kinetic energy referred to the CB
$\boldsymbol{x}$	=	full airship inertial velocity vector represented in ABC frame, $[\boldsymbol{V}^T, \omega^T]^T \in \Re^{6\times 1}$
$\boldsymbol{x}_a$	=	vector of air velocity, $[\boldsymbol{V}_a^T, \omega_a^T]^T = (\boldsymbol{x} - \boldsymbol{x}_w) \in \Re^{6\times 1}$
$\boldsymbol{x}_w$	=	full wind velocity vector represented in ABC frame, $[\boldsymbol{V}_w^T, \omega_w^T]^T \in \Re^{6\times 1}$
$\boldsymbol{x}^c \in \Re^{6\times 1}$	=	full inertial velocity vector of the c.g.
$\boldsymbol{x}^0$	=	full inertial velocity vector of the CB, nearly equal to $\boldsymbol{x} \in \Re^{6\times 1}$
δ	=	prefix indicating a small perturbation of a given variable around its nominal value
$\delta\boldsymbol{x}_h$	=	state vector of linearized lateral model, $[\delta v, \delta p, \delta r, \delta\phi]^T \in \Re^{4\times 1}$
$\delta\boldsymbol{x}_v$	=	state vector of linearized longitudinal model, $[\delta u, \delta w, \delta q, \delta\theta]^T \in \Re^{4\times 1}$
ϕ, θ, ψ	=	roll, pitch, and yaw Euler angles associated with transformation $\boldsymbol{S}$
ω	=	vector of airship angular speed represented in ABC frame, $[p, q, r]^T \in \Re^{3\times 1}$

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ω_a = vector of angular airspeed, $\omega - \omega_w \in \Re^{3\times 1}$
ω_w = vector of wind angular speed represented in ABC frame, $[p_w, q_w, r_w]^T \in \Re^{3\times 1}$

Subscripts

a = apparent for masses; airspeed for velocities
B = buoyancy
h = horizontal plane (lateral mode)
v = vertical plane (longitudinal mode); virtual
w = wind

Superscripts

c = regarding the c.g.
0 = regarding the CV

I. Introduction

THE modeling of a vehicle moving in a fluid is an important part of the analysis and design of vehicles such as airships and submersibles. Some usual difficulties arise in the computation of this kind of motion equations due to the mass of the vehicle and the mass of the displaced fluid being of the same order of magnitude. For example, when the vehicle is accelerated in a fluid, additional forces are required to increase the kinetic energy contained in the fluid. This effect appears as an apparent increase in the mass of the vehicle and is often referred to as added mass.

The important work of Lamb[1] was the first to analyze the motion of a rigid vehicle in a perfect fluid. Lamb showed that the kinetic energy of the fluid can be expressed as a quadratic form involving the three translational and three rotational components of the vehicle velocity. In his approach, the calculation of the fluid pressure effects on the surface of the vehicle is solved by treating solid and fluid as a single dynamic system.

The derivations of Lamb were used by Jones and Williams[2] to generate the equations of motion of airships and by Lewis et al.[3] to model the dynamics of a remotely operated underwater vehicle. They were also applied to simulate the dynamics of the YEZ-2A airship by Nippress and Gomes[4] and Gomes.[5]

In a recent paper, Thomasson[6] reviewed the work of Lewis et al.[3] that proposed a new formulation of the equations of motion of a rigid vehicle in an unsteady heavy fluid. By the treatment of the effects of the inertial and added masses as separate functions of the inertial and relative velocities, Thomasson[6] derived a common set of motion equations to be applied generically to submersibles, airships, and airplanes. In his new formulation, the effects of fluid motion and velocity gradients were also considered.

Following the formulation of Thomasson,[6] we write here the equation of motion of an airship and consider separately, in the Lagrangian energy approach, the three kinetic energy terms associated with 1) the energy of the vehicle motion itself, 2) the energy of the air around the airship due to the relative velocities, and finally 3) the energy added to the buoyancy air (displaced by the airship volume). The first term is proportional to the vehicle inertial velocity, whereas the second and third terms are proportional to the relative velocity (or airspeed).

An equivalent formulation is then introduced that allows the translation of the airship equations of motion from the usual Lagragian approach into a Newton's second law formulation. In the derivation of the force and moment equations, the velocity of the air mass results in a new term, wind-induced force and torque, that is not present in the formulation of Gomes.[5] This wind-induced term may be regarded as the inertial time derivative of the apparent momentum of the buoyancy air as viewed by the airship. It expresses the tendency of the airship to be dragged away by the wind.

In particular, for an airship in a constant wind, it is shown that the wind-induced force is proportional to the airship angular speed, which results in a change in the damping characteristics of the motion. Obviously, the actual influence of the wind-induced force on the airship dynamics depends on the magnitude of the other terms of the dynamic equation, particularly the kinematic and aerodynamic forces. The longitudinal and lateral dynamics of the airship motion were analyzed for a given range in the wind speed (considering a constant low airspeed). For the analysis of the wind influence on the airship dynamics, the model of the AURORA project AS800 airship[7,8] was considered.

This paper is organized as follows. The airship dynamic equation of motion is developed in Sec. II. In this section, the mass matrix, with its virtual and apparent terms, is presented, and the wind-induced force and torque terms are derived. In Sec. III, the influence of a constant wind on the longitudinal and lateral dynamics of the airship motion is studied through a detailed analysis of the wind-induced force and torque terms. Section IV verifies the results of Sec. III using the model of a real airship and considering an air-hovering situation (zero airspeed) in a constant wind. Finally, Sec. V presents the main conclusions of the work and the Appendix presents some changing references between inertial frame and rigid body frame.

II. Equations of Motion of an Airship

A. Assumptions

In the attempt to establish a workable mathematical model of the airship flight, a number of considerations have to be taken into account.

First, the airship displaces a very large volume of air; the mass and inertia properties of this air become significant, that is, the airship behaves as if it had mass and moments of inertia substantially higher than those associated to its own physical construction.

Second, three kinds of masses and inertia matrices must be considered: the mass and inertia $(m, \boldsymbol{J})$ of the vehicle itself, the mass and inertia $(m_B, \boldsymbol{J}_B)$ of the buoyancy air (corresponding to the air displaced by the total volume of the airship), and the virtual mass and inertia $(\boldsymbol{M}_v, \boldsymbol{J}_v)$ (which may be regarded as the mass of air around the airship and displaced with the relative motion of the airship in the air).

Third, the airship mass changes in flight due to ballonet deflation or inflation: here, however, it is assumed to be slow varying with the associated time derivatives equal to zero.

Fourth, to accommodate the constantly changing c.g., the airship motion has to be referenced to a system of orthogonal axes fixed in the vehicle with the origin at the center of volume (CV) (Fig. 1). The CV is also assumed to coincide with the gross center of buoyancy (CB).

Fifth, the airship is assumed to be a rigid body, and the aeroelastic effects are ignored.

Sixth, the airframe is symmetric about its vertical (XZ) plane such that both the CV and the c.g. lie in the plane of symmetry.

Last, the Earth is assumed as flat and taken as an inertial frame.

B. Dynamic Equations of Motion

The equations describing the atmospheric flight of an aircraft are often derived from Newton's second law of classical mechanics, expressed in the inertial frame, north, east, down (NED):

$$\left.\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t}\right|_{\mathrm{NED}} = \boldsymbol{F}, \qquad \left.\frac{\mathrm{d}\boldsymbol{H}}{\mathrm{d}t}\right|_{\mathrm{NED}} = \boldsymbol{T} \tag{1}$$

where $(\boldsymbol{F}, \boldsymbol{T})$ are the total external force and torque, which are commonly described as the sum of gravitational $(\boldsymbol{F}_g, \boldsymbol{T}_g)$, aerodynamic $(\boldsymbol{F}_a, \boldsymbol{T}_a)$, and propulsion $(\boldsymbol{F}_p, \boldsymbol{T}_p)$ forces.

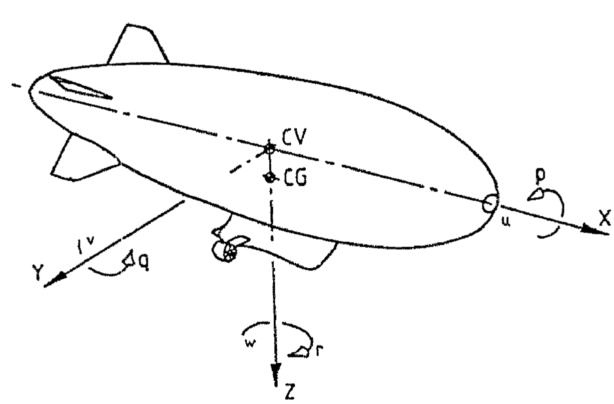

Fig. 1 Local frame ABC.

When the displaced fluid mass is not negligible, as is the case for airships or balloons, the equations of motion are usually derived from the Lagrangian approach.[1,6]

To express the kinetic energy, let the motion of the airship be described by its inertial velocity $\boldsymbol{x} = [\boldsymbol{V}^T, \omega^T]^T$, a six-degree-of-freedom vector including the inertial linear speed $\boldsymbol{V}$ and angular speed ω. Let the surrounding air be described by an inertial wind velocity $\boldsymbol{x}_w = [\boldsymbol{V}_w^T, \omega_w^T]^T$. The airship has, thus, a relative air velocity equal to the difference of the previous two ($\boldsymbol{x}_a = \boldsymbol{x} - \boldsymbol{x}_w$). The total kinetic energy W is then obtained as a sum,

$$W = W^c + W_B^0 + W_v^0 \tag{2}$$

that accounts for the vehicle motion W^c, for the kinetic energy added to the buoyancy air W_B^0 (displaced by the airship volume), and also for the energy due to the virtual mass W_v^0. These terms are detailed as follows.

The kinetic energy of the vehicle depends on the inertial velocity,

$$W^c = \tfrac{1}{2}\boldsymbol{x}^{cT}\overline{\boldsymbol{M}^c}\boldsymbol{x}^c \tag{3}$$

where $\overline{\boldsymbol{M}^c}$ is the generalized mass matrix, including the scalar mass m and inertial matrix $\boldsymbol{J}$ of the vehicle. The inertial term is normally referred to the c.g. $\boldsymbol{C}$ and to its velocity $\boldsymbol{x}^c$. The other two terms in Eq. (2) that correspond to the air replaced and around the airship volume are referred to the CB $\boldsymbol{O}$ and to its velocity $\boldsymbol{x} \doteq \boldsymbol{x}^0$.

In the case of the buoyancy air, to cancel the contribution of the air mass motion, the inertial kinetic energy W_{Bi}^0 must first be subtracted, and then only the extra kinetic energy W_{Ba}^0 is added[6]:

$$W_B^0 = -W_{Bi}^0 + W_{Ba}^0 = -\tfrac{1}{2}\boldsymbol{x}^T\overline{\boldsymbol{M}_B}\boldsymbol{x} + \tfrac{1}{2}\boldsymbol{x}_a^T\overline{\boldsymbol{M}_B}\boldsymbol{x}_a \tag{4}$$

where $\overline{\boldsymbol{M}_B}$ is the generalized mass matrix of the buoyancy air (vehicle total volume filled with air).

Finally, for the last term in Eq. (2), which may be regarded as the kinetic energy of the air moving around the airship due to the relative velocities, the notions of virtual mass and virtual inertia $\overline{\boldsymbol{M}_v}$ were introduced by Lamb[1]:

$$W_v^0 = \tfrac{1}{2}\boldsymbol{x}_a^T\overline{\boldsymbol{M}_v}\boldsymbol{x}_a \tag{5}$$

C. Inclusion of Virtual Inertia into Momentum

The notions of virtual masses and inertia may also be introduced into the formulation of Newton's second law (1) through the equivalent definition of a total linear and angular momentum.

Following the definition of the kinetic energy (2), the linear momentum is then defined as the sum of the vehicle momentum, proportional to the inertial speed of the c.g., plus the momentum added to the displaced buoyancy air, depending on the inertial and relative speed of the CB, plus the virtual mass momentum proportional to the airspeed of the CB:

$$\boldsymbol{p} = m\boldsymbol{V}^c + (m_B\boldsymbol{V}_a - m_B\boldsymbol{V}) + \boldsymbol{M}_v\boldsymbol{V}_a \tag{6}$$

Likewise, the total angular momentum around the origin is the sum of the inertial angular momentum of the vehicle around point O, plus a term corresponding to the buoyancy air, plus a virtual term proportional to the relative angular speed,

$$\boldsymbol{H} = (\boldsymbol{J}\omega + \boldsymbol{OC} \times m\boldsymbol{V}) + (\boldsymbol{J}_B\omega_a - \boldsymbol{J}_B\omega) + \boldsymbol{J}_v\omega_a \tag{7}$$

where the superscripts were omitted because all variables are relative to the origin $O = \mathrm{CV}$.

Such a definition for the linear and angular momenta corresponds to the standard case where the total virtual matrix is reduced to a block diagonal matrix, with a virtual mass matrix $\boldsymbol{M}_v$ and a virtual inertia matrix $\boldsymbol{J}_v$:

$$\overline{\boldsymbol{M}_v} = \begin{bmatrix} \boldsymbol{M}_v & 0 \\ 0 & \boldsymbol{J}_v \end{bmatrix} \tag{8}$$

D. Derivation of the Force Equation

Once the linear momentum has been defined, the *ABC* velocity and the wind velocity are introduced:

$$\boldsymbol{p} = m(\boldsymbol{V} - \boldsymbol{OC} \times \omega) + m_B(\boldsymbol{V}_a - \boldsymbol{V}) + \boldsymbol{M}_v\boldsymbol{V}_a \tag{9}$$

The airspeed $V_a = V - V_w$ may be replaced introducing the wind speed,

$$\boldsymbol{p} = m\boldsymbol{V} - m\boldsymbol{OC} \times \omega + \boldsymbol{M}_v(\boldsymbol{V} - \boldsymbol{V}_w) - m_B\boldsymbol{V}_w \tag{10}$$

resulting in the final formulation of the linear momentum,

$$\boldsymbol{p} = \boldsymbol{M}_a\boldsymbol{V} - m\boldsymbol{OC} \times \omega - \boldsymbol{M}_{Ba}\boldsymbol{V}_w \tag{11}$$

where the apparent mass of the vehicle $\boldsymbol{M}_a = m\boldsymbol{I}_3 + \boldsymbol{M}_v$ and the apparent mass of the buoyancy air $\boldsymbol{M}_{Ba} = m_B\boldsymbol{I}_3 + \boldsymbol{M}_v$ were used.

The time derivative of the momentum (1) is first applied in the inertial frame:

$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = \frac{\mathrm{d}(\boldsymbol{M}_a\boldsymbol{V} - m\boldsymbol{OC} \times \omega - \boldsymbol{M}_{Ba}\boldsymbol{V}_w)}{\mathrm{d}t} = \boldsymbol{F} \tag{12}$$

$$\frac{\mathrm{d}\boldsymbol{M}_a\boldsymbol{V}}{\mathrm{d}t} - m\left[\boldsymbol{OC} \times \frac{\mathrm{d}\omega}{\mathrm{d}t} + \frac{\mathrm{d}(\boldsymbol{OC})}{\mathrm{d}t} \times \omega\right] - \frac{\mathrm{d}\boldsymbol{M}_{Ba}\boldsymbol{V}_w}{\mathrm{d}t} = \boldsymbol{F} \tag{13}$$

and then transported into the local frame,

$$\begin{aligned}(\boldsymbol{M}_a\dot{\boldsymbol{V}} + \omega \times \boldsymbol{M}_a\boldsymbol{V}) - m\boldsymbol{OC} \times \dot{\omega} - m(\omega \times \boldsymbol{OC}) \times \omega \\ - (\boldsymbol{M}_{Ba}\dot{\boldsymbol{V}}_w + \omega \times \boldsymbol{M}_{Ba}\boldsymbol{V}_w) = \boldsymbol{F}\end{aligned} \tag{14}$$

leading to the final form of the force equation,

$$[\boldsymbol{M}_a \quad -m\boldsymbol{OC}\times]\dot{\boldsymbol{x}} = \boldsymbol{F} + \boldsymbol{F}_k + \boldsymbol{F}_w \tag{15}$$

where two new force terms appear, respectively, the inertial kinematics force and the wind-induced force:

$$\boldsymbol{F}_k = -m\omega \times (\omega \times \boldsymbol{OC}) - \omega \times \boldsymbol{M}_a\boldsymbol{V} \tag{16}$$

$$\boldsymbol{F}_w = \boldsymbol{M}_{Ba}\dot{\boldsymbol{V}}_w + \omega \times \boldsymbol{M}_{Ba}\boldsymbol{V}_w \tag{17}$$

E. Derivation of the Angular Equation

As for the earlier case, the wind angular speed is introduced into the definition of the angular momentum,

$$\boldsymbol{H} = \boldsymbol{J}_a\omega + \boldsymbol{OC} \times m\boldsymbol{V} - \boldsymbol{J}_{Ba}\omega_w \tag{18}$$

where the apparent vehicle and buoyancy air inertial matrices, $\boldsymbol{J}_a = \boldsymbol{J} + \boldsymbol{J}_v$ and $\boldsymbol{J}_{Ba} = \boldsymbol{J}_B + \boldsymbol{J}_v$, respectively, have been introduced.

Therefore, the angular momentum equation (1) becomes

$$\begin{aligned}(\boldsymbol{J}_a\dot{\omega} + \omega \times \boldsymbol{J}_a\omega) + [\boldsymbol{OC} \times m\dot{\boldsymbol{V}} + \omega \times (\boldsymbol{OC} \times m\boldsymbol{V})] \\ - (\boldsymbol{J}_{Ba}\dot{\omega}_w + \omega \times \boldsymbol{J}_{Ba}\omega_w) = \boldsymbol{T}\end{aligned} \tag{19}$$

or in matrix form,

$$[m\boldsymbol{OC}\times \quad \boldsymbol{J}_a]\dot{\boldsymbol{x}} = \boldsymbol{T} + \boldsymbol{T}_k + \boldsymbol{T}_w \tag{20}$$

where two new torque terms appear, the inertial kinematics torque and the wind-induced torque, respectively,

$$\boldsymbol{T}_k = -\omega \times \boldsymbol{J}_a\omega - \omega \times (\boldsymbol{OC} \times m\boldsymbol{V}) \tag{21}$$

$$\boldsymbol{T}_w = \boldsymbol{J}_{Ba}\dot{\omega}_w + \omega \times \boldsymbol{J}_{Ba}\omega_w \tag{22}$$

The final matricial equation for the motion of the airship in wind is obtained from Eqs. (15) and (20):

$$\overline{\boldsymbol{M}_a}\dot{\boldsymbol{x}} = \overline{\boldsymbol{F}_a} + \overline{\boldsymbol{F}_g} + \overline{\boldsymbol{F}_p} + \overline{\boldsymbol{F}_k} + \overline{\boldsymbol{F}_w} \tag{23}$$

with the generalized apparent mass matrix defined by

$$\overline{\boldsymbol{M}_a} = \begin{bmatrix} \boldsymbol{M}_a & -m\boldsymbol{OC}\times \\ m\boldsymbol{OC}\times & \boldsymbol{J}_a \end{bmatrix} \tag{24}$$

and the generalized forces produced by aerodynamics, $\overline{\boldsymbol{F}_a}$; gravity, $\overline{\boldsymbol{F}_g}$; propulsion, $\overline{\boldsymbol{F}_p}$; kinematics, $\overline{\boldsymbol{F}_k}$; and wind, $\overline{\boldsymbol{F}_w}$.

F. Apparent Mass Matrix

With regard to the generalized apparent mass matrix (24), the apparent mass and inertia matrices are given by

$$M_a = mI_3 + M_v, \qquad J_a = J + J_v \tag{25}$$

and the virtual mass and inertia matrices are usually taken as[1]

$$M_v = \begin{bmatrix} -X_{\dot{u}} & 0 & 0 \\ 0 & -Y_{\dot{v}} & 0 \\ 0 & 0 & -Z_{\dot{w}} \end{bmatrix}, \qquad J_v = \begin{bmatrix} L_{\dot{p}} & 0 & 0 \\ 0 & M_{\dot{q}} & 0 \\ 0 & 0 & N_{\dot{r}} \end{bmatrix} \tag{26}$$

where $X_{\dot{u}}$, $Y_{\dot{v}}$, $Z_{\dot{w}}$ and $L_{\dot{p}}$, $M_{\dot{q}}$, $N_{\dot{r}}$ are Lamb's virtual mass and inertia terms, which are all proportional to the buoyancy mass m_B.

G. Inclusion of Gravity and Attitude

The gravity acceleration is given in the Earth NED frame as a constant vector $a_g = [0, 0, g]^T$. Thus, in the local frame (Fig. 1), the gravity force, which adds the weight force, applied at the c.g., and the buoyancy force, applied at the CB or CV, is a function of the transformation matrix S from Earth frame to local frame:

$$\overline{F_g} = \begin{bmatrix} F_g \\ T_g \end{bmatrix} = \begin{bmatrix} S(m - m_B)a_g \\ OC \times Sma_g \end{bmatrix} \tag{27}$$

The kinematic relationship between attitude and angular rates brings a new differential equation,

$$\dot{S} = -\omega \times S \tag{28}$$

which may be expressed in terms of the Euler angles (ϕ, θ, ψ) and their derivatives.

H. Linearized Model

The complexity of the nonlinear dynamic equations justifies the search for a linear simplified version, also important to analyze and evaluate the characteristics of the airship dynamics. The linearization of the dynamic equations (23) is performed for trimmed conditions around equilibrium, which is commonly that of a horizontal straight flight, without wind incidence.

In such conditions, the equations are written for a perturbation vector δx of the states around the equilibrium value x_0 and the perturbed input δu around the trimmed value u_0, resulting in the matricial dynamic equation

$$\delta\dot{x} = A\delta x + B\delta u \tag{29}$$

in the absence of disturbances (deterministic case).

One important result of the linearization approach is the separation of two independent (decoupled) motions: the motion in the vertical plane, called longitudinal, and the motion in the horizontal plane, called lateral.

The linearized models, that is, the dynamic matrix A and the input matrix B, depend on the trim point chosen for the linearization and, in particular, on the airspeed V_{t0} and altitude h_0 chosen. The existence of a constant wind component is also to be considered.

For the longitudinal case, in the vertical plane, the state vector considered to evaluate the dynamic characteristics is $\delta x_v = [\delta u, \delta w, \delta q, \delta\theta]^T$ and the input vector is given by δu_v. The longitudinal dynamic equation is then given by

$$\delta\dot{x}_v = A_v\delta x_v + B_v\delta u_v \tag{30}$$

For the lateral case, in the horizontal plane, the state vector considered is $\delta x_h = [\delta v, \delta p, \delta r, \delta\phi]^T$ and the input vector is given by δu_h. (Here the trim values are generally null.) The lateral dynamic equation is, thus, given by

$$\delta\dot{x}_h = A_h\delta x_h + B_h\delta u_h \tag{31}$$

The longitudinal and lateral linearized dynamic models are used in the following section to evaluate the damping and natural frequency of the system under a dominant wind influence.

III. Influence of Wind on the Airship Dynamics

Because of their large volume and low density, airships are enormously affected by the wind. In this section, the dynamic motion equation (23) is further analyzed for the condition where the wind-induced force $\overline{F_w}$ is dominant over the other force components, allowing for the investigation of the wind influence on the airship dynamics. Initially, two hypothetical extreme case suppositions for an airship are presented and analyzed. Then, the wind-induced force $\overline{F_w}$ is derived as a function of the airship geometry, wind velocity, and airship angular speed. Finally, the dynamics of the airship in a hovering neutral flight (null airspeed velocity) for constant longitudinal or lateral wind is analyzed.

A. Extreme Cases of Airships

We investigate here two hypothetical situations for an airship. The first one considers the airship replaced by a fictitious volume of pure air ($m = m_B$), and the second one supposes that the airship has a negligible buoyancy mass compared to the vehicle mass ($m \gg m_B$).

1. *Volume of Pure Air Case*

First, we suppose that the airship is replaced by a fictitious volume of pure air, in which case the scalar mass is exactly the buoyancy mass ($m = m_B$). In this situation, the mass and inertia matrices are equal to the mass and inertia buoyancy matrices ($M_a = M_{Ba}$, $J_a = J_{Ba}$); CB and c.g. are the same, yielding $OC = 0$; and the aerodynamic, propulsion, and gravity forces are all null. Therefore, the dynamic equation (23) reduces to

$$\begin{aligned} \overline{M_a}\dot{x} &= \begin{bmatrix} M_a & -mOC\times \\ mOC\times & J_a \end{bmatrix} \begin{bmatrix} \dot{V} \\ \dot{\omega} \end{bmatrix} \\ &= \overline{F_k} + \overline{F_w} \Rightarrow \begin{bmatrix} M_{Ba} & 0 \\ 0 & J_{Ba} \end{bmatrix} \begin{bmatrix} \dot{V} \\ \dot{\omega} \end{bmatrix} \\ &= \begin{bmatrix} -\omega \times M_{Ba}V \\ -\omega \times J_{Ba}\omega \end{bmatrix} + \begin{bmatrix} M_{Ba}\dot{V}_w + \omega \times M_{Ba}V_w \\ J_{Ba}\dot{\omega}_w + \omega \times J_{Ba}\omega_w \end{bmatrix} \end{aligned} \tag{32}$$

which accepts the obvious intuitive solution ($V = V_w$, $\omega = \omega_w$), which states that the volume of air is dragged and moving along with the wind or surrounding air.

2. *Aircraft Case*

On the other hand, if the buoyancy mass is negligible as compared to the vehicle mass, as is the case of an aircraft, then the virtual mass may also be neglected. Because we have now $m \gg m_B$ and $\overline{M_a} \simeq \overline{M}$, the components of matrix $\overline{M_v}$ tend to be much smaller than those of matrix $\overline{M}$. Thus, the wind-induced force ($\overline{F_w}$) given in Eq. (17) may be neglected when compared to the kinematic force $\overline{F_k}$ given in Eq. (16):

$$\overline{F_k} + \overline{F_w} \simeq \overline{F_k} \simeq \begin{bmatrix} -m\omega \times (\omega \times OC) - \omega \times mV \\ -\omega \times J\omega - \omega \times (OC \times mV) \end{bmatrix} \tag{33}$$

In terms of dynamic characteristics, this is equivalent to omitting the influence of the wind speed or to neglecting it as compared to the airspeed and inertial speed or $V \simeq V_a$ and $\omega \simeq \omega_a$.

In this case of a negligible buoyancy mass, there is no need to consider the CB as the origin of the local frame, and instead it may be assumed located at the c.g. This would be equivalent to considering $OC = 0$ in the dynamic equations, leading to a further simplification of Eqs. (23) and (24), which are reduced to

$$\begin{bmatrix} M & 0 \\ 0 & J \end{bmatrix} \begin{bmatrix} \dot{V} \\ \dot{\omega} \end{bmatrix} = \overline{F_a} + \overline{F_g} + \overline{F_p} + \begin{bmatrix} -\omega \times mV \\ -\omega \times J\omega \end{bmatrix} \tag{34}$$

Finally, because the aerodynamic forces only depend on the air velocity V_a, the dynamic behavior of the model is taken as a function of the air velocity, which is the common assumption in aircraft flight analysis.

B. Airship in Neutral Flight

In the case of an airship in neutral flight, for which the weight is exactly matched by the buoyancy force ($m = m_B$), the virtual mass cannot be omitted and neither can the apparent buoyancy mass. It is interesting then to study the influence of the wind-induced force. To do so, we first derive the wind-induced force $\overline{\boldsymbol{F}_w}$ explicitly as a function of the airship geometry, wind velocity, and angular speed, and then the dynamics of the airship is analyzed for constant longitudinal or lateral wind incidences.

Consider the general case as presented in the dynamic equation (23); the influence of wind is directly expressed by the wind-induced term:

$$\overline{\boldsymbol{F}_w} = \begin{bmatrix} \boldsymbol{M}_{Ba}\dot{\boldsymbol{V}}_w + \omega \times \boldsymbol{M}_{Ba}\boldsymbol{V}_w \\ \boldsymbol{J}_{Ba}\dot{\omega}_w + \omega \times \boldsymbol{J}_{Ba}\omega_w \end{bmatrix} \tag{35}$$

This force term may be regarded as the inertial time derivative of the apparent momentum of the buoyancy air as viewed by the airship. It expresses the tendency of the airship to be dragged away by the wind, somewhat like in the preceding extreme case where the airship is a volume of pure air.

C. Airship in Constant Wind

To analyze the influence of wind on the airship dynamics, let us consider the typical case where wind may be assumed as constant in the Earth NED frame with components $\boldsymbol{V}_w^{\mathrm{NED}} = [V_N, V_E, V_D]^T$. We also suppose null angular speed components for the wind.

The local components of this constant wind depend on the transformation matrix $\boldsymbol{S}$,

$$\boldsymbol{V}_w = [u_w, v_w, w_w]^T = \boldsymbol{S}\boldsymbol{V}_w^{\mathrm{NED}} \tag{36}$$

as well as its time derivative,

$$\dot{\boldsymbol{V}}_w = \dot{\boldsymbol{S}}\boldsymbol{V}_w^{\mathrm{NED}} = -\omega \times \boldsymbol{S}\boldsymbol{V}_w^{\mathrm{NED}} = -\omega \times \boldsymbol{V}_w \tag{37}$$

The wind-induced force (17) is then given by

$$\boldsymbol{F}_w = -\boldsymbol{M}_{Ba}\omega \times \boldsymbol{V}_w + \omega \times \boldsymbol{M}_{Ba}\boldsymbol{V}_w = (-\boldsymbol{M}_{Ba}\omega \times + \omega \times \boldsymbol{M}_{Ba})\boldsymbol{V}_w \tag{38}$$

If the airship geometry is approximated by an ellipsoid of revolution, as is usually done, then the virtual mass matrix may be taken as diagonal, as well as the apparent buoyancy mass matrix,

$$\boldsymbol{M}_{Ba} = \boldsymbol{M}_v + m_B\boldsymbol{I}_3 = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & b \end{bmatrix} + m_B\boldsymbol{I}_3 \tag{39}$$

where a and b are virtual mass elements that are function of the ellipsoid geometry. The wind-induced force from Eq. (38) is then given by

$$\boldsymbol{F}_w = (a - b)\begin{bmatrix} 0 & r & -q \\ r & 0 & 0 \\ -q & 0 & 0 \end{bmatrix}\begin{bmatrix} u_w \\ v_w \\ w_w \end{bmatrix} = (a - b)\begin{bmatrix} v_w r - w_w q \\ u_w r \\ -u_w q \end{bmatrix} \tag{40}$$

where the local ABC components of the constant wind have been used.

A first result from the preceding expression is that for a volume with spherical geometry (perhaps nearly the case of a balloon) that has an isotropic virtual mass ($a = b$) and, thus, an isotropic apparent buoyancy mass; the wind-induced force is null ($\boldsymbol{F}_w = 0$), and the dynamics is then insensitive to the wind.

Some further conclusions may be drawn from Eq. (40):

1) The wind-induced force is proportional to the angular speed and will result in a change in the damping characteristics of the motion.

2) The longitudinal component of the wind, u_w, produces a lateral and vertical force function of the yaw and pitch rates, respectively, in agreement with the usual separation between longitudinal motion, described by the longitudinal state $\delta\boldsymbol{x}_v = [\delta u, \delta w, \delta q, \delta\theta]^T$, and lateral motion, described by the lateral state $\delta\boldsymbol{x}_h = [\delta v, \delta p, \delta r, \delta\phi]^T$.

3) Likewise, the vertical wind component w_w produces a longitudinal force function of the pitch rate, both belonging to the longitudinal motion.

4) On the contrary, the lateral wind component v_w results in a longitudinal force function of the yaw rate and, thus, introduces a coupling between longitudinal and lateral modes.

Obviously the influence of the wind-induced force $\boldsymbol{F}_w$ depends on the magnitude of the other terms of the dynamic equation, particularly the kinematics term $\boldsymbol{F}_k$, which is a function of the inertial velocity, and the aerodynamic term $\boldsymbol{F}_a$, which is a function of the air velocity. That is, wind will mostly affect the airship dynamics when both these two velocities, and also the propulsive (and control) forces, are low.

To reach some quantified results, it is interesting to analyze the dynamics of the airship motion when varying the wind (and inertial) speed considering a low constant airspeed or, in other words, under constant low aerodynamic forces.

D. Dynamics of an Airship in Hovering Flight

Here, the dynamic equation (23) is explicitly derived considering the case of negligible aerodynamic forces. The resulting equation is then separated into the corresponding longitudinal and lateral ones, and with consideration of a given structure for the mass submatrices, they are written in the equivalent linearized form. Finally, two cases of wind incidence (longitudinal or lateral wind) are considered, and the dynamics of the linearized equations are analyzed for each situation.

Let us consider, initially, that the airship is air hovering, that is, the airspeed is zero and, thus, the aerodynamic forces vanish. Let us also consider the propulsive (and control) forces as zero. In such a hovering neutral flight, the dynamics of the airship is essentially like a pendulum with two degrees of freedom, exhibiting two oscillatory modes, one in pitch and one in roll.

The dynamic equation (23) is then reduced to

$$\overline{\boldsymbol{M}_a}\dot{\boldsymbol{x}} = \overline{\boldsymbol{F}_g} + \overline{\boldsymbol{F}_k} + \overline{\boldsymbol{F}_w} \tag{41}$$

If only small perturbations are considered around the equilibrium state, the vector of velocities derivative can be written as $\dot{\boldsymbol{x}} = [\dot{\boldsymbol{V}}^T, \dot{\omega}^T]^T = [\delta\dot{u}, \delta\dot{v}, \delta\dot{w}, \delta\dot{p}, \delta\dot{q}, \delta\dot{r}]^T$. Also, with zero attitude, the transformation matrix, used in the evaluation of $\overline{\boldsymbol{F}_g}$ [Eq. (27)], may be approximated to first order by

$$\boldsymbol{S} = \begin{bmatrix} 1 & \delta\psi & -\delta\theta \\ -\delta\psi & 1 & \delta\phi \\ \delta\theta & -\delta\phi & 1 \end{bmatrix} \tag{42}$$

Because airspeed is zero, the equilibrium inertial speed is equal to the wind speed ($u = u_w$, $v = v_w$, $w = w_w$), and considering that the trim values for the angular velocities are null, then we have $p = \delta p$, $q = \delta q$, $r = \delta r$. Now, from Eqs. (27), (16), (21), and (40), the dynamic equation (41) can be written as

$$\overline{\boldsymbol{M}_a}\dot{\boldsymbol{x}} = mg\begin{bmatrix} 0 \\ 0 \\ 0 \\ -a_z\delta\phi \\ -a_z\delta\theta \\ a_x\delta\phi \end{bmatrix} + \begin{bmatrix} (m+b)(v_w\delta r - w_w\delta q) \\ (m+b)w_w\delta p - (m+a)u_w\delta r \\ (m+a)u_w\delta q - (m+b)v_w\delta p \\ ma_z u_w\delta r - ma_x(v_w\delta q + w_w\delta r) \\ mv_w(a_z\delta r + a_x\delta p) \\ -ma_z(u_w\delta p + v_w\delta q) + ma_x w_w\delta p \end{bmatrix} + (a - b)\begin{bmatrix} v_w\delta r - w_w\delta q \\ u_w\delta r \\ -u_w\delta q \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{43}$$

Finally, we reorder the vector elements of the preceding equation by grouping the longitudinal (δu, δw, δq) and the lateral (δv, δp, δr) variables as

$$\begin{bmatrix} \boldsymbol{M}_{av} & 0 \\ 0 & \boldsymbol{M}_{ah} \end{bmatrix} \begin{bmatrix} \delta\dot{u} \\ \delta\dot{w} \\ \delta\dot{q} \\ \delta\dot{v} \\ \delta\dot{p} \\ \delta\dot{r} \end{bmatrix} = \begin{bmatrix} (m+a)(v_w\delta r - w_w\delta q) \\ (m+b)(u_w\delta q - v_w\delta p) \\ m(-ga_z\delta\theta + a_z v_w\delta r + a_x v_w\delta p) \\ (m+b)(w_w\delta p - u_w\delta r) \\ m(-ga_z\delta\phi + a_z u_w\delta r - a_x v_w\delta q - a_x w_w\delta r) \\ m(ga_x\delta\phi - a_z u_w\delta p - a_z v_w\delta q + a_x w_w\delta p) \end{bmatrix} \tag{44}$$

where the matrices $\boldsymbol{M}_{av} \in \Re^{3\times 3}$ and $\boldsymbol{M}_{ah} \in \Re^{3\times 3}$ are obtained from the corresponding elements on matrix $\overline{\boldsymbol{M}_a} \in \Re^{6\times 6}$ as follows. From Eqs. (24–26), and from consideration of the diagonal structure for the virtual mass matrix presented in Eq. (39), the generalized apparent mass matrix results in

$$\overline{\boldsymbol{M}_a} = \begin{bmatrix} m+a & 0 & 0 & 0 & ma_z & 0 \\ 0 & m+b & 0 & -ma_z & 0 & ma_x \\ 0 & 0 & m+b & 0 & -ma_x & 0 \\ 0 & -ma_z & 0 & J_x + L_{\dot{p}} & 0 & -J_{xz} \\ ma_z & 0 & -ma_x & 0 & J_y + M_{\dot{q}} & 0 \\ 0 & ma_x & 0 & -J_{xz} & 0 & J_z + N_{\dot{r}} \end{bmatrix} \tag{45}$$

Then, the longitudinal and lateral mass matrices $\boldsymbol{M}_{av}$ and $\boldsymbol{M}_{ah}$ are given by

$$\boldsymbol{M}_{av} = \begin{bmatrix} m+a & 0 & ma_z \\ 0 & m+b & -ma_x \\ ma_z & -ma_x & J_y + M_{\dot{q}} \end{bmatrix}$$

$$\boldsymbol{M}_{ah} = \begin{bmatrix} m+b & -ma_z & ma_x \\ -ma_z & J_x + L_{\dot{p}} & -J_{xz} \\ ma_x & -J_{xz} & J_z + N_{\dot{r}} \end{bmatrix} \tag{46}$$

E. Constant Wind in the Airship Vertical Plane

If wind is in the airship vertical plane ($u_w \neq 0$, $v_w = 0$, $w_w \neq 0$), then the dynamics may be split into the longitudinal and the lateral motion. This is possible because the substitution of $v_w = 0$ in Eq. (44) leads to a set of equations where the longitudinal motion described by the three first lines in Eq. (44) are only dependent on the longitudinal variables, or

$$\begin{bmatrix} \boldsymbol{M}_{av} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \delta\dot{u} \\ \delta\dot{w} \\ \delta\dot{q} \\ \delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} -(m+a)w_w\delta q \\ (m+b)u_w\delta q \\ -mga_z\delta\theta \\ \delta q \end{bmatrix} \tag{47}$$

and in the same way, the lateral motion described by the three last lines in Eq. (44) appear as function of the lateral variables only, or

$$\begin{bmatrix} \boldsymbol{M}_{ah} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \delta\dot{v} \\ \delta\dot{p} \\ \delta\dot{r} \\ \delta\dot{\phi} \end{bmatrix} = \begin{bmatrix} (m+b)(w_w\delta p - u_w\delta r) \\ m(-ga_z\delta\phi + a_z u_w\delta r - a_x w_w\delta r) \\ m(ga_x\delta\phi - a_z u_w\delta p + a_x w_w\delta p) \\ \delta p \end{bmatrix} \tag{48}$$

with the inclusion of the pitch and roll attitude angles $\delta\theta$ and $\delta\phi$.

The preceding dynamic equations may be written in state-space form, respectively, as

$$\begin{bmatrix} \delta\dot{u} \\ \delta\dot{w} \\ \delta\dot{q} \\ \delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} \boldsymbol{M}_{av} & 0 \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 & -(m+a)w_w & 0 \\ 0 & 0 & (m+b)u_w & 0 \\ 0 & 0 & 0 & -mga_z \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \delta u \\ \delta w \\ \delta q \\ \delta\theta \end{bmatrix} \tag{49}$$

$$\begin{bmatrix} \delta\dot{v} \\ \delta\dot{p} \\ \delta\dot{r} \\ \delta\dot{\phi} \end{bmatrix} = \begin{bmatrix} \boldsymbol{M}_{ah} & 0 \\ 0 & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} 0 & (m+b)w_w & -(m+b)u_w & 0 \\ 0 & 0 & m(a_z u_w - a_x w_w) & -mga_z \\ 0 & -m(a_z u_w - a_x w_w) & 0 & mga_x \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta v \\ \delta p \\ \delta r \\ \delta\phi \end{bmatrix} \tag{50}$$

If we consider the usual case where the c.g. is nearly below the CB (or $a_x \cong 0$ and $a_z \gg a_x$), a further simplification may be obtained. In addition, when it is assumed that $a_x \cong 0$ and $J_{xz} \cong 0$, it is possible to derive a standard form for the inverse of the mass matrices $\boldsymbol{M}_{av}$ and $\boldsymbol{M}_{ah}$:

$$\boldsymbol{M}_{av}^{-1} = \begin{bmatrix} m+a & 0 & ma_z \\ 0 & m+b & -ma_x \\ ma_z & -ma_x & J_y + M_{\dot{q}} \end{bmatrix}^{-1} \simeq \begin{bmatrix} n_{uu} & 0 & n_{uq} \\ 0 & n_{ww} & 0 \\ n_{uq} & 0 & n_{qq} \end{bmatrix} \tag{51}$$

$$\boldsymbol{M}_{ah}^{-1} = \begin{bmatrix} m+b & -ma_z & ma_x \\ -ma_z & J_x + L_{\dot{p}} & -J_{xz} \\ ma_x & -J_{xz} & J_z + N_{\dot{r}} \end{bmatrix}^{-1} \simeq \begin{bmatrix} n_{vv} & n_{vp} & 0 \\ n_{vp} & n_{pp} & 0 \\ 0 & 0 & n_{rr} \end{bmatrix} \tag{52}$$

Therefore, the linearized longitudinal [Eq. (49)] and lateral [Eq. (50)] free dynamics are finally written as

$$\delta\dot{\boldsymbol{x}}_v = \boldsymbol{A}_v\delta\boldsymbol{x}_v = \begin{bmatrix} 0 & 0 & -n_{uu}(m+a)w_w & -n_{uq}mga_z \\ 0 & 0 & n_{ww}(m+b)u_w & 0 \\ 0 & 0 & -n_{uq}(m+a)w_w & -n_{qq}mga_z \\ 0 & 0 & 1 & 0 \end{bmatrix}\delta\boldsymbol{x}_v \tag{53}$$

$$\delta\dot{\boldsymbol{x}}_h = \boldsymbol{A}_h\delta\boldsymbol{x}_h = \begin{bmatrix} 0 & n_{vv}(m+b)w_w & -n_{vv}(m+b)u_w + n_{vp}ma_z u_w & -n_{vp}mga_z \\ 0 & n_{vp}(m+b)w_w & -n_{vp}(m+b)u_w + n_{pp}ma_z u_w & -n_{pp}mga_z \\ 0 & -n_{rr}ma_z u_w & 0 & n_{rr}mga_x \\ 0 & 1 & 0 & 0 \end{bmatrix}\delta\boldsymbol{x}_h \tag{54}$$

where $\delta\boldsymbol{x}_v = [\delta u, \delta w, \delta q, \delta\theta]^T$ and $\delta\boldsymbol{x}_h = [\delta v, \delta p, \delta r, \delta\phi]^T$.

For the longitudinal dynamics, it can be seen from the dynamic matrix $\boldsymbol{A}_v$ in Eq. (53) that the system will present two poles at the origin, related to the speed components (u, w). The third line of the system corresponds to the pitch oscillation (q, θ) and has a second-order characteristic polynomial,

$$\Delta(s) = s^2 + n_{uq}(m+a)w_w s + n_{qq}mga_z \tag{55}$$

which reveals a constant natural frequency $\omega_\theta = \sqrt{(n_{qq} m g a_z)}$ and a damping that is linearly dependent on the vertical wind speed component, whereas the longitudinal component has no effect.

In the lateral dynamics equation (54), the speed component v is not present, which results in a pole at the origin. The other states give a third-order characteristic polynomial:

$$\Delta(s) = s^3 - n_{vp}(m+b)w_w s^2 + mg\{n_{pp}a_z - n_{rr}a_x u_w[n_{pp}ma_z - n_{vp}(m+b)]\}s + u_w[n_{pp}ma_z - n_{vp}(m+b)]n_{rr}ma_z u_w \tag{56}$$

yielding a real pole and a pair of conjugates, corresponding to the roll oscillation (p, ϕ). With no wind, the natural frequency of this oscillation is $\omega_\phi = \sqrt{(n_{pp} m g a_z)}$. As for the longitudinal dynamics, the effect of wind is a damping and frequency change of the roll oscillation, coupled with a yaw component.

F. Constant Lateral Wind

For the case presented earlier, with the wind in the vertical plane $(u_w \neq 0, v_w = 0, w_w \neq 0)$, the longitudinal and lateral dynamics were shown to be decoupled, as given by Eqs. (47) and (48). However, in the case of lateral wind $(u_w = 0, v_w \neq 0, w_w = 0)$, coupling appears between longitudinal and lateral modes in a sixth-order system. This can can be seen from Eq. (44), with $u_w = 0$ and $w_w = 0$:

$$\begin{bmatrix} \boldsymbol{M}_{av} & 0 \\ 0 & \boldsymbol{M}_{ah} \end{bmatrix} \begin{bmatrix} \delta\dot{u} \\ \delta\dot{w} \\ \delta\dot{q} \\ \delta\dot{v} \\ \delta\dot{p} \\ \delta\dot{r} \end{bmatrix} = \begin{bmatrix} (m+a)v_w\delta r \\ -(m+b)v_w\delta p \\ m(-ga_z\delta\theta + a_z v_w \delta r + a_x v_w \delta p) \\ 0 \\ m(-ga_z\delta\phi - a_x v_w \delta q) \\ m(ga_x\delta\phi - a_z v_w \delta q) \end{bmatrix} \tag{57}$$

Clearly it is not possible to split the equations in two sets of independent dynamics because the longitudinal motion (three first lines) depends both on the longitudinal and lateral variables, as well as the lateral motion given by the three last lines. Together with this coupling between longitudinal and lateral motions, the presence of lateral wind will obviously affect the poles of both oscillatory modes.

IV. Case Study for Real Airship Model

To verify the preceding results and illustrate the wind influence on the airship dynamics, the model of a real airship was used. The AS800[9–11] model is a nonrigid, 9-m-long, 2.25-m-diam, 24-m^3 airship equipped with two vectored engines, four X-shaped control surfaces at the stern, and an additional dc tail motor. The X-shaped deflection surfaces of the tail generate the equivalent rudder, elevator, and aileron commands of the classical + tail.

The pole location of the longitudinal and lateral modes were computed, with constant airspeed and with wind speed varying between 0 and 5 m/s. The poles were computed numerically using small perturbation inputs in the complete nonlinear model represented by Eq. (23).

A. Air-Hovering in Constant Wind

A first situation corresponds to the hovering case discussed earlier, with zero airspeed. A horizontal wind was first considered, and the results are presented in Fig. 2. The longitudinal poles (Fig. 2a) are clearly insensitive, whereas the frequency of the roll oscillation increases with the wind speed (Fig. 2b).

Figure 3 presents the case of a lateral wind (from the right of the airship). The lateral poles (Fig. 3b) do not show changes for the given wind speed range, and only the pitch oscillation frequency (Fig. 3a) is lowered when the wind speed increases. A closer inspection at

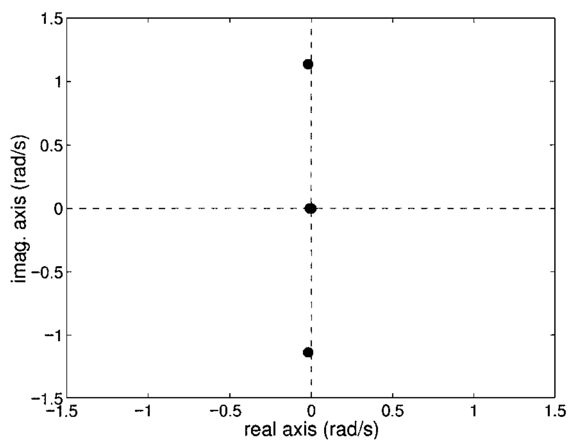

Fig. 2a Longitudinal poles for hovering and varying longitudinal wind speed (from 0 to 5 m/s).

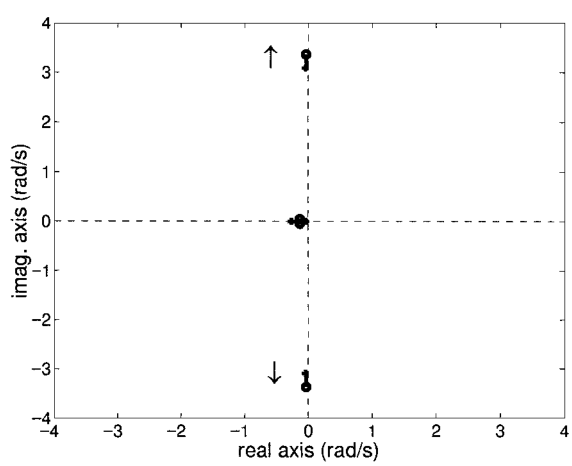

Fig. 2b Lateral poles for the same situation.

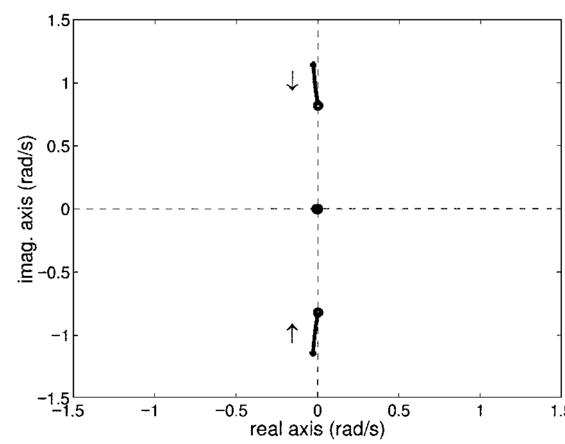

Fig. 3a Longitudinal poles for hovering and varying lateral wind speed (from 0 to 5 m/s).

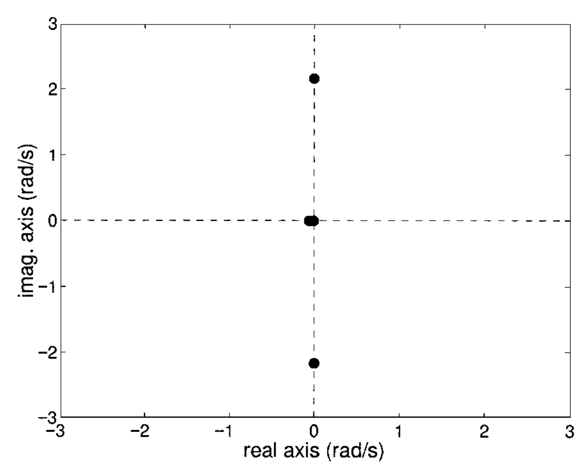

Fig. 3b Lateral poles for the same situation.

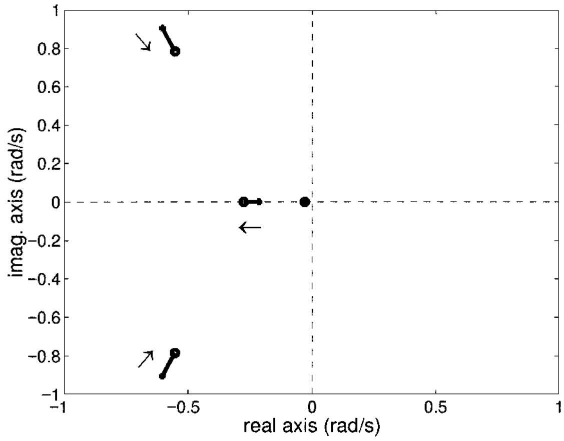

Fig. 4a Longitudinal poles for 2 m/s airspeed and varying longitudinal wind speed (from 0 to 5 m/s).

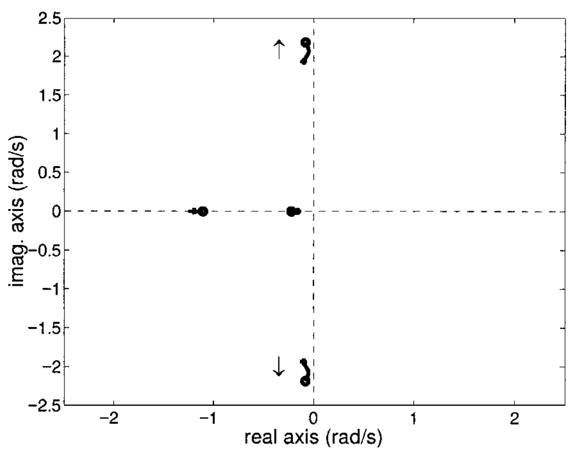

Fig. 4b Lateral poles for the same situation.

Fig. 3 shows that the pitch damping is very sensitive, with a tendency toward instability for higher wind speeds. In fact, the increase in the oscillatory behavior for higher lateral wind speeds is related to the aerodynamic forces that will result from such a condition. When a lateral wind is applied to the airship, a tendency of longitudinal backward movement will appear due to the forces acting on the tail fins. As a consequence, the control inversion and the small velocities involved will result in strong oscillations in the pitch mode. In practice the instability will never occur because there will always be some amount of aerodynamic damping forces. The conclusion is that the lateral wind speed reduces the damping in longitudinal mode in an indirect way, whereas direct influence of the wind is more visible in the reduction of the frequency of the oscillation as shown in (Fig. 3a).

B. Near Hovering with 2-m/s Airspeed

The case of a low airspeed was then considered with an airspeed of 2 m/s. For such an airspeed, the aerodynamic forces already introduce some damping in the oscillation poles but inertial and wind-induced forces are still predominant.

The influence of a longitudinal wind component is presented in Fig. 4. When compared to the previous cases, all of the poles have been globally taken to the left and are, thus, more damped. The effect on the longitudinal poles (Fig. 4a) is mostly to reduce the frequency of the pitch oscillation. On the lateral poles (Fig. 4b), the roll oscillation is still very little damped, and the frequency is somewhat increased when increasing the wind speed.

The case of a lateral wind component is presented in Fig. 5. Here the lateral poles (Fig. 5b) are quite insensitive to wind (similar to the hovering case), whereas the pitch oscillation (Fig. 5a) is changed, exhibiting a slightly lower damping ratio and a lower frequency when the wind speed is increased.

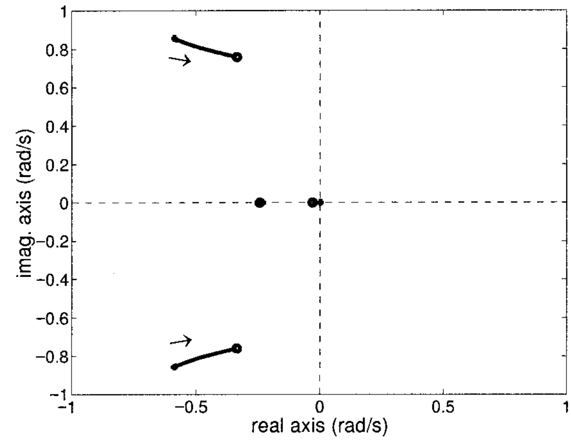

Fig. 5a Longitudinal poles for 2 m/s airspeed and varying lateral wind speed (from 0 to 5 m/s).

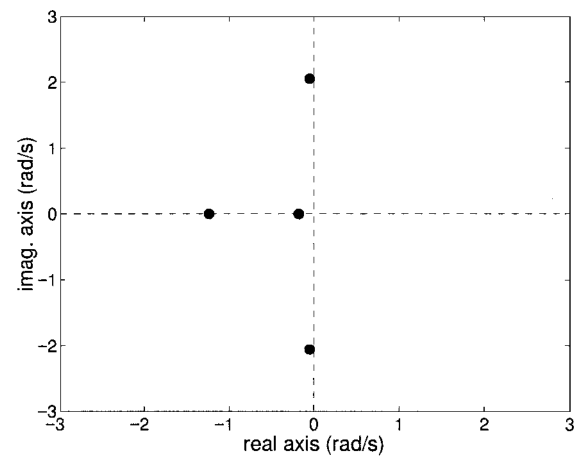

Fig. 5b Lateral poles for the same situation.

V. Conclusions

A new formulation of the equations of motion of an airship was derived allowing the analysis of the wind influence on the airship dynamics. Initially, the equations of motion are written in the Lagrangian approach, with consideration of the three kinetic energy terms associated with 1) the energy of the vehicle motion itself, 2) the energy of the air around the airship due to the relative velocities, and 3) the energy added to the buoyancy air. An equivalent formulation was introduced allowing the translation of the equations of motion from the Lagrangian approach into Newton's second law formulation. In the derivation of the force and moment equations, the velocity of the air mass resulted in a new term, wind-induced force and torque, representing the inertial time derivative of the apparent momentum of the buoyancy air as viewed by the airship.

When the airship geometry approximated by an ellipsoid of revolution is considered, the wind-induced force and torque terms were explicitly derived, and their contribution to the longitudinal and lateral dynamics of the airship motion was analyzed. To exemplify the results, the model of a real airship was used to verify the wind influence on the dynamics, for a given range of wind speed and a constant low airspeed, showing how the wind-induced forces do affect the damping of the oscillatory modes.

Appendix: Changing References

The time derivative is defined in the inertial NED frame. The time derivative from inertial frame to *ABC* frame introduces the Coriolis acceleration

$$\frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t}_{\mathrm{NED}} = \frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t}_{ABC} + \omega \times \boldsymbol{V} = \dot{\boldsymbol{V}} + \omega \times \boldsymbol{V}$$

Following the assumption of a rigid body, the linear speed of the c.g. (point C) is related to the linear speed of the CV (point O) through the angular speed:

$$\boldsymbol{V}^c = \boldsymbol{V}^0 + \omega \times \boldsymbol{OC} = \boldsymbol{V} - \boldsymbol{OC} \times \omega$$

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